Boğaziçi University

ECONFIN - MATHEMATICS WAIVER EXAM

Solutions

04/01/2018

NAME:

1. (30 pts) Consider the following matrices.

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & 2 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 0 & 2 \end{bmatrix}, \; D = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$

Compute each of the the following. If it is not computable, explain why.

(a) $A\ B\ C$

Solution:

$$A\ B\ C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 21 & 16 \\ 8 & 8 \end{bmatrix}$$

(b) $(A^T)\ (B^T)\ C$

Solution:

Not computable. The number of columns in B^T is not equal to the number of rows in C.

(c) Inverse of D

Solution:

$$D^{-1} = \frac{1}{det(D)} \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & -3/2 \\ 0 & 1 \end{bmatrix}$$

2. (20 pts) Differentiate the following functions with respect to x.

(a) $f(x) = \frac{x^3+2x-1}{1-x^3}$

Solution:

$$f'(x) = \frac{(3x^2+2)(1-x^3)-(x^3+2x-1)(-3x^2)}{(1-x^3)^2} = \frac{3x^2-3x^5+2-2x^3+3x^5+6x^3-3x^2}{(1-x^3)^2}$$

$$= \frac{4x^3+2}{(1-x^3)^2} = 2\frac{2x^3+1}{(1-x^3)^2}$$

(b) $f(x) = (5x^2-3)^2(\ln(x^3))$

Solution:

$$f'(x) = \ln(x^2)\cdot[2(5x^2-3)\cdot 10x] + 3x^2\frac{1}{x^3}\cdot(5x^2-3)^2$$

$$= (5x^2-3)[\ln(x^2)\cdot 20x + \frac{3}{x}(5x^2-3)]$$

3. (30 pts) Solve the following integrals. Use integration by parts if needed.

(a) $\int(\frac{1}{x} + 2e^{-x} + x^4)dx$

Solution:

$\int(\frac{1}{x} + 2e^{-x} + x^4)dx = \ln(x) - 2e^{-x} + \frac{x^5}{5} + C$

(b) $\int_1^e 2x\ln(x)dx$

Solution:

$\int_1^e 2x\ln(x)dx = \int_1^e udv$

where $u = \ln(x)$ and $dv = 2xdx$.

So, $du = (1/x)dx$ and $v = \int 2xdx = x^2$

Then, by integration by parts we get

$\int_1^e 2x\ln(x)dx = \int_1^e udv = uv - \int_1^e vdu = [x^2\ln(x)]_1^e - \int_1^e x^2\frac{1}{x}dx = [e^2\ln(e) - 1^2\ln(1)] - \int_1^e xdx$

Then,

$\int_1^e 2x\ln(x)dx = e^2 - \int_1^e xdx = e^2 - [\frac{x^2}{2}]_1^e = e^2 - [e^2/2 - (1/2)] = \frac{e^2+1}{2}$

4. (10 pts) Find the extreme values of the following functions and determine whether they are maxima or minima.

$f(x, y) = x^2 + xy + 2y^2 + 3$

Solution:

$\frac{\partial f}{\partial x} = 2x + y = 0$

$\frac{\partial f}{\partial y} = x + 4y = 0$

$\frac{\partial^2 f}{\partial x^2} = f_{xx} = 2$

$\frac{\partial^2 f}{\partial y^2} = f_{yy} = 4$

$\frac{\partial^2 f}{\partial x \partial y} = f_{xy} = 1$

Then, we get $y = -2x$. which implies and $x + 4y = x + 4(-2x) = -7x = 0$ that is $x = 0$ and then $y = 0$.

Since $f_{xx}f_{yy} - f_{xy}^2 = 2 \cdot 4 - 1 = 7 > 0$ and $f_{xx} = 2 > 0$ and $f_{yy} = 4 > 0$, we get $d^2 f > 0$. Thus, the point $(x, y) = (0, 0)$ is a relative minimum.

5. (10 pts) For the function below, use the first derivative test to find when relative extrema occur. Specify as relative maxima/minima.

   $f(x) = -10x^2 + 40x + 28$

   Solution:

   $f'(x) = -20x + 40 = 0$ implies $x = 2$.

   Since $f''(x = 2) = -20 < 0$ $x = 2$ is relative maximum.